Chapter 3
Baseband pulse and digital signaling
3.2 Pulse amplitude modulation
3.2 Pulse amplitude modulation

- **Basis** ---- The sampling theorem
- **PAM.** The amplitude of the carrier pulse varies with the analog baseband signal.

**Characteristics of PAM signal**
- Pulse-type signal, its amplitude denotes the analog information.
- Satisfied with sampling theorem
- PAM’s bandwidth is wider than that of analog waveform

Two classes of PAM signals

- Natural Sampling (Gating)
- Instantaneous Sampling
3.2 Pulse amplitude modulation

3.2.1 Natural Sampling (Gating)

Generation of PAM with natural sampling

\[ W_s(t) = w(t)s(t) \]

Analog bilateral switch
If \( w(t) \) is an analog waveform bandlimited to \( B \) Hertz, the PAM signal that uses natural sampling (gating) is

\[
\mathcal{w}_s(t) = w(t) s(t)
\]

where

\[
s(t) = \sum_{k = -\infty}^{\infty} \prod \left( \frac{t - kT_s}{\tau} \right)
\]
The spectrum for a naturally sampled PAM signal is

\[ W_s(f) = F[w_s(t)] = d \sum_{n=-\infty}^{\infty} \frac{\sin n\pi d}{n\pi d} W(f - nf_s) \]  

(3-3)

Where \( W(f) = F[w(t)] \) is the spectrum of the original unsampled waveform.

The spectrum of PAM signal with natural sampling is a function of the spectrum of the analog input waveform.
Example.
The case of an input waveform that has a rectangular spectrum, where the duty cycle of the switching waveform is
\[ d = \frac{\tau}{T_s} = \frac{1}{3} \]
And the sampling rate is
\[ f_s = 4B \]
Demodulation of PAM signal

- **Method 1 ---- Low-pass filter**
- **Method 2 ---- Product detection**

**Figure 3-4** Demodulation of a PAM signal (naturally sampled).
3.2.2 Instantaneous sampling

**Generation**

by using a sample-and-hold type of electronic circuit

\[
W(t) \rightarrow \delta(t-KT_s) \rightarrow h(t) = \prod \left( \frac{t}{\tau} \right) \rightarrow W_s(t)
\]
Characteristic

At $t=kT_s$, the sampling values $w(kT_s)$ determine the amplitude of the flat-top rectangular pulses.
If w(t) is a analog waveform bandlimited to B hertz, the instantaneous sampled PAM signal is given by

\[ w_s(t) = \sum_{k=-\infty}^{\infty} w(kT_s)h(t - kT_s) \]

where \( h(t) \) denotes the sampling-pulse shape,

\[ h(t) = \prod \left( \frac{t}{\tau} \right) = \begin{cases} 1, & |t| < \tau / 2 \\ 0, & |t| > \tau / 2 \end{cases} \]

where \( \tau \leq T_s = 1/f_s \) and \( f_s \geq 2B \)
The spectrum for a Flat-top PAM signal is

\[
W_s(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} W(f - kf_s)
\]

where

\[
H(f) = F[h(t)] = \tau \left( \sin \frac{\pi tf}{\pi tf_s} \right)
\]
Spectrum of Flat-top PAM

(a) Magnitude Spectrum of Input Analog Waveform

(b) Magnitude Spectrum of PAM (flat-top sampling), $\tau/T_s = 1/3$ and $f_s = 4B$
Demodulation of flat-top PAM

- **Method 1 ---- Low-pass filter**

\[
\frac{W_s(f)}{1/H(f)} \rightarrow \frac{W_s(f)}{LPF} \rightarrow W_s(f)
\]

**Note:**

1: Equalization filter ------ reduce the high frequency loss
2: decreasing \( \tau \), The pulse width \( \tau \) is called aperture.
Prefilter is needed before the multiplier to compensate for the spectral loss due to the aperture effect.

**Note:**

Prefilter is needed before the multiplier to compensate for the spectral loss due to the aperture effect.

Where \( B < f_{co} < f_s - B \)
Comparison -- in time domain

Comparison in time domain: waveform of PAM signal

Fig 3-1 PAM Signal with natural sampling

Fig 3-5 PAM signal with flat-top sampling
Comparison -- in frequency domain

Spectrum of PAM signal

Fig 3-3 The Spectrum of natural sampling PAM

\[ W_s(f) = d \sum_{n=-\infty}^{\infty} \frac{\sin n \pi d}{n \pi d} W(f - nf_s) \]

Fig 3-6 The Spectrum of Flat-top PAM

\[ W_s(f) = \frac{1}{T_s} H(f) \sum_{n=-\infty}^{\infty} W(f - nf_s) \]

\[ = d \sum_{n=-\infty}^{\infty} \left( \frac{\sin \pi \frac{f}{f_s}}{\pi \frac{f}{f_s}} \right) W(f - nf_s) \]
The bandwidth required for the transmission of PAM is much larger than that of the original analog signal.

Noise performance of the PAM system can never be better than that of the analog signal. Not very good for long-distance transmission.

Provide a means for converting an analog signal to a PCM signal. Time-division multiplexing etc.
3.3 Pulse code modulation
Pulse Code Modulation

Analog waveform

Sampling
- t1: -4.88 v
- t2: +21.43 v

Quantizing
- -5 v
- 21 v

Coding & Streaming
- 1000 0101 = -5
- 0001 0101 = 21

..., 0001 0101, 1000 0101, ...
Pulse Code Modulation

Three basic operations

- Sampling
- Quantizing
- Encoding

PCM transmitter (A/D conversion)

\[ x(t) \xrightarrow{\text{sampler}} x_n \xrightarrow{\text{quantizer}} \tilde{x}_n \xrightarrow{\text{encoder}} s(t) \]

\[ \tilde{x}(t) \xrightarrow{\text{Low-pass filter}} \]

\[ \text{decoder} \]
Quantizing

Approximating the analog sample values by using finite number of levels

- Uniform quantizing
- Quantizing error
- Quantizing noise
Pulse Code Modulation

Encoding

- The quantized analog sample values are replaced by n-bit binary code

E.g. three-bit Gray code for M=8 levels

\[ M = 2^n \]

<table>
<thead>
<tr>
<th>Quantized Sample Voltage</th>
<th>Gray Code Word (PCM output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+7</td>
<td>110</td>
</tr>
<tr>
<td>+5</td>
<td>111</td>
</tr>
<tr>
<td>+3</td>
<td>101</td>
</tr>
<tr>
<td>+1</td>
<td>100</td>
</tr>
<tr>
<td>-1</td>
<td>000</td>
</tr>
<tr>
<td>-3</td>
<td>001</td>
</tr>
<tr>
<td>-5</td>
<td>011</td>
</tr>
<tr>
<td>-7</td>
<td>010</td>
</tr>
</tbody>
</table>
The bit rate of PCM signal is

\[ R = n f_s \]

**Example.** Design of a PCM signal for telephone system

Assume:

An analog audio voice-frequency (VF) telephone signal band: 300Hz ~ 3400 Hz

The minimum sampling frequency is \( 2 \times 3.4 = 6.8 \) ksample/sec. actually, using sampling frequency of 8 ksample/sec.

**Bit rate:** \[ R = f_s \text{ (samples/s)} \times n \text{ (bits/sample)} \]

\[ = 8 \text{ k sample/s} \times 8 \text{ bits/sample} = 64 \text{ kbps} \]
The bandwidth of (serial) binary PCM waveforms depends on:

- The bit rate
- The waveform pulse shape used to represent the data

\[ B_{PCM} = R = n f_s \]

<table>
<thead>
<tr>
<th>Number of quantizer levels, ( M )</th>
<th>Length of the PCM, ( n )(bit)</th>
<th>Bandwidth of PCM signal (the first null bandwidth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2B</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4B</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>6B</td>
</tr>
<tr>
<td>256</td>
<td>8</td>
<td>16B</td>
</tr>
<tr>
<td>......</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Effects of noise

Two main effects produce noise or distortion:

- **Bit errors** in the recovered PCM signal. (channel noise, improper channel filtering, ISI etc.)

- **Quantizing noise** that is caused by the M-step quantized at PCM transmitter.
Effects of noise

PCM transmitter (A/D conversion)

\[ x(t) \rightarrow \text{sampler} \rightarrow x_n \rightarrow \text{quantizer} \rightarrow \tilde{x}_n \rightarrow \text{encoder} \rightarrow \text{channel} \rightarrow 0101110\ldots \rightarrow \text{decoder} \rightarrow \tilde{x}(t) \rightarrow \text{Low-pass filter} \]
Under certain assumptions, the ratio of the recover analog peak signal power to the total average noise power is:

\[
\left( \frac{S}{N} \right)_{pk\ out} = \frac{3M^2}{1 + 4(M^2 - 1)P_e}
\]

The ratio of the average signal power to the average noise power is

\[
\left( \frac{S}{N} \right)_{out} = \frac{M^2}{1 + 4(M^2 - 1)P_e}
\]
Effects of noise

◆ If $P_e = 0$ (no ISI), the peak SNR resulting from only quantizing errors is

$$\left( \frac{S}{N} \right)_{pk\ out} = 3M^2$$

◆ The average SNR due only to quantizing error is

$$\left( \frac{S}{N} \right)_{out} = M^2$$

◆ 6-dB rule

$$\left( \frac{S}{N} \right)_{dB} = 6.02n + \alpha$$

$\alpha = 4.77$ for the peak SNR,
$\alpha = 0$ for the average SNR.
This equation points out the significant performance characteristic for PCM:

An additional 6-dB improvement in SNR is obtained for each bit added to the PCM word.

Assumptions:

1. No bit errors
2. The input signal level is large enough to range over a significant number of quantizing levels
## Performance

<table>
<thead>
<tr>
<th>Number of Quantizer Levels Used, $M$</th>
<th>Length of the PCM Word, $n$ (bits)</th>
<th>Bandwidth of PCM Signal (First Null Bandwidth)$^a$</th>
<th>Recovered Analog Signal-Power-to-Quantizing-Noise Power Ratios (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>$2B$</td>
<td>$(S/N)<em>{pk,out}$: 10.8, $(S/N)</em>{out}$: 6.0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$4B$</td>
<td>$(S/N)<em>{pk,out}$: 16.8, $(S/N)</em>{out}$: 12.0</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>$6B$</td>
<td>$(S/N)<em>{pk,out}$: 22.8, $(S/N)</em>{out}$: 18.1</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>$8B$</td>
<td>$(S/N)<em>{pk,out}$: 28.9, $(S/N)</em>{out}$: 24.1</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
<td>$10B$</td>
<td>$(S/N)<em>{pk,out}$: 34.9, $(S/N)</em>{out}$: 30.1</td>
</tr>
<tr>
<td>64</td>
<td>6</td>
<td>$12B$</td>
<td>$(S/N)<em>{pk,out}$: 40.9, $(S/N)</em>{out}$: 36.1</td>
</tr>
<tr>
<td>128</td>
<td>7</td>
<td>$14B$</td>
<td>$(S/N)<em>{pk,out}$: 46.9, $(S/N)</em>{out}$: 42.1</td>
</tr>
<tr>
<td>256</td>
<td>8</td>
<td>$16B$</td>
<td>$(S/N)<em>{pk,out}$: 52.9, $(S/N)</em>{out}$: 48.2</td>
</tr>
<tr>
<td>512</td>
<td>9</td>
<td>$18B$</td>
<td>$(S/N)<em>{pk,out}$: 59.0, $(S/N)</em>{out}$: 54.2</td>
</tr>
<tr>
<td>1,024</td>
<td>10</td>
<td>$20B$</td>
<td>$(S/N)<em>{pk,out}$: 65.0, $(S/N)</em>{out}$: 60.2</td>
</tr>
<tr>
<td>2,048</td>
<td>11</td>
<td>$22B$</td>
<td>$(S/N)<em>{pk,out}$: 71.0, $(S/N)</em>{out}$: 66.2</td>
</tr>
<tr>
<td>4,096</td>
<td>12</td>
<td>$24B$</td>
<td>$(S/N)<em>{pk,out}$: 77.0, $(S/N)</em>{out}$: 72.2</td>
</tr>
<tr>
<td>8,192</td>
<td>13</td>
<td>$26B$</td>
<td>$(S/N)<em>{pk,out}$: 83.0, $(S/N)</em>{out}$: 78.3</td>
</tr>
<tr>
<td>16,384</td>
<td>14</td>
<td>$28B$</td>
<td>$(S/N)<em>{pk,out}$: 89.1, $(S/N)</em>{out}$: 84.3</td>
</tr>
<tr>
<td>32,768</td>
<td>15</td>
<td>$30B$</td>
<td>$(S/N)<em>{pk,out}$: 95.1, $(S/N)</em>{out}$: 90.3</td>
</tr>
<tr>
<td>65,536</td>
<td>16</td>
<td>$32B$</td>
<td>$(S/N)<em>{pk,out}$: 101.1, $(S/N)</em>{out}$: 96.3</td>
</tr>
</tbody>
</table>

$^a$ $B$ is the absolute bandwidth of the input analog signal.
In a communications-quality audio system, an analog voice-frequency (VF) signal with a bandwidth of 3200 Hz is converted into a PCM signal by sampling at 7000 samples/s and by using a uniform quantizer with 64 steps. The PCM binary data are transmitted over a noisy channel to a receiver that has a bit error rate (BER) of $10^{-4}$.

What is the null bandwidth of the PCM signal if a polar line code is used?

What is the average SNR of the recovered analog signal at the receiving end?
Nonuniform Quantizing

- Characteristic voice analog signal
  - Nonuniform amplitude distribution
  - The granular quantizing noise will be a serious problem if uniform quantizing is used.

- Solution: nonuniform quantizing

- Nonuniform Quantizing: a variable step size is used
Method:
- passing the analog signal through a compression (nonlinear) amplifier and then into a PCM circuit that uses uniform quantizer.

μ-Law and A-Low
Nonuniform Quantizing

μ-Law

\[ |w_2(t)| = \frac{\ln(1 + \mu |w_1(t)|)}{\ln(1 + \mu)} \quad 0 \leq |w_1(t)| \leq 1 \]

(a) M=8 Quantizer Characteristic

Compression quantizer characteristic

Uniform quantizer characteristic
Nonuniform Quantizing

\[ |w_2(t)| = \begin{cases} 
\frac{A|w_1(t)|}{\ln(1 + \mu)}, & 0 \leq |w_1(t)| \leq \frac{1}{A} \\
\frac{1 + \ln(A|w_1(t)|)}{1 + \ln A}, & \frac{1}{A} \leq |w_1(t)| \leq 1 
\end{cases} \]
In practice, the smooth nonlinear characteristics of $\mu$-Law and A-Low are approximated by piecewise linear chords.
Following 6-dB law:

\[
\left( \frac{S}{N} \right)_{dB} = 6.02n + \alpha
\]

where

\[
\alpha = 4.77 - 20 \log \left( \frac{V}{x_{rms}} \right)
\]

Uniform quantizing

\[
\alpha = 4.77 - 20 \log [\ln (1 + \mu)]
\]

\[-\text{Law companding}

\[
\alpha = 4.77 - 20 \log [1 + \ln A]
\]

A-Law companding
Comparison of output SNR
3.4 Digital signaling
Introduction

- **Baseband signals**
  - the signals involved in the analog-to-digital conversion

- **Bandpass signals**
  - The signals produced by using baseband digital signals to modulate a carrier.
In this section, we will answer the following questions:

- How do we mathematically represent the waveform for a digital signal?

- How do we estimate the bandwidth of the waveform?
- The voltage (or current) waveforms for digital signals can be expressed as an orthogonal series with a finite number of terms $N$.

$$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t)$$

- $w_k$ represents the digital data. Binary signaling, multilevel signaling.
- $\varphi_k(t)$ are $N$ orthogonal functions that give the waveform for a digital signal.
- $N$ is the number of dimensions required to describe the waveform.
The orthogonal function representation of digital signaling corresponds to the orthogonal vector space represented by

\[ W = \sum_{j=1}^{N} W_j \phi_j \]

A short-hand notation for the vector \( w \) is given by a row vector

\[ W = ( w_1, w_2, \ldots, w_N ) \]
**baud rate and bit rate**

**Data rate:**

- The baud (symbol rate)

\[ D = \frac{N}{T_0} \text{ symbol/s} \]

Where \( N \) is the number of dimensions used in \( T_0 \) s.

- The bit rate

\[ R = \frac{n}{T_0} \text{ bits/s} \]

Where \( n \) is the number of data bits sent in \( T_0 \) s.

- For the case of the binary signal, \( n = N \), \( R = D \).

- For the case of the multilevel signal, \( n \neq N \), \( R \neq D \).
Bandwidth estimation

◆ The lower bound for the bandwidth of the waveform \( w(t) \) is:

\[
B \geq \frac{N}{2T_0} = \frac{1}{2} D \quad (\text{Hz}) \quad (3-32)
\]

◆ If the \( \phi_k(t) \) are of the \( \sin(x)/x \) type, then

\[
B = D/2 \quad (\text{Hz})
\]

◆ otherwise

\[
B > D/2 \quad (\text{Hz})
\]

Equation (3-32) is useful for predicting the bandwidth of digital signals, especially when the exact bandwidth of the signal is difficult to calculate.
The data rate of the binary encoded PCM signal is

\[ D = R = n f_s \]

The bandwidth of the binary encoded PCM waveform is bounded by

\[ B_{PCM} \geq \frac{1}{2} R = \frac{1}{2} n f_s = n B \]

The minimum bandwidth of \( \frac{1}{2} R = \frac{1}{2} n f_s \) is obtained only when \((\sin x)/x\) type pulse is used to generate the PCM waveform.
The bandwidth of (serial) binary PCM waveforms depends on:
- The bit rate
- The waveform pulse shape used to represent the data

For rectangular pulse, the first null bandwidth is

\[ B_{PCM} = R = nf_s \]

Example. the result for the case of the minimum sampling:

<table>
<thead>
<tr>
<th>Number of quantizer levels, M</th>
<th>Length of the PCM, n(bit)</th>
<th>Bandwidth of PCM signal (the first null bandwidth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2B</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4B</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>6B</td>
</tr>
<tr>
<td>256</td>
<td>8</td>
<td>16B</td>
</tr>
<tr>
<td>......</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.4.3 Binary signaling

- The orthogonal series coefficients $w_k$ take on binary values

**Example 3-3**

- A digital source can produce $M=256$ distinct messages. Each message is represented by $n=8$-bit binary words, assume that it takes $T_0=8$ ms to transmit one message, a particular message corresponding to the code word $01001110$ is to be transmitted.

  - $w_1=0, w_2=1, w_3=0, w_4=0, w_5=1, w_6=1, w_7=1, w_8=0$

- **Case 1.** Rectangular pulse orthogonal function
- **Case 2.** $\sin(x)/x$ pulse orthogonal function
example

a. Rectangular pulse shape, $T_b=1$ ms

b. $\sin(x) / x$ pulse shape, $T_b=1$ ms
Binary signaling

Note:

- When the rectangular pulse shape is used, the digital source information is transmitted via a binary waveform.

- When the $\sin(x)/x$ pulse shape is used, the digital source information is transmitted via an analog waveform.
Multilevel signaling

**Question:** the lower-bound bandwidth is 
\[ B = \frac{N}{(2T_0)} \], can the bandwidth be made smaller?

**Thinking:** if \( N \) could be reduced, the bandwidth could be made smaller.

- \( N \) can be reduced by letting the \( w_k \) takes on \( L > 2 \) possible values.

- When \( w_k \) has \( L > 2 \) possible values, the resulting waveform is called **multilevel signal.**
Example 3-4 \( L=4 \) multilevel signal

The source of example 3-3 will be encoded into \( L=4 \) multilevel signal, and the message will be sent in \( T_0=8 \) ms.

\[
\text{Binary signal } \, w_1(t) \quad \text{\( R \) bit/s} \quad \text{\( \ell \) bit D/A transfer} \quad \text{L-level waveform signal } \, w_2(t) \quad \text{D symbol/s=\( R/\ell \), \( R \) bit/s}
\]

\( L=2^\ell \)
### L=4 multilevel signal

One possible encoding scheme for an $\ell=2$ bit:

<table>
<thead>
<tr>
<th>Binary input ($\ell=2$ bits)</th>
<th>Output level ($v$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>+3</td>
</tr>
<tr>
<td>10</td>
<td>+1</td>
</tr>
<tr>
<td>00</td>
<td>-1</td>
</tr>
<tr>
<td>01</td>
<td>-3</td>
</tr>
</tbody>
</table>

For the binary code word 01001110, the $w_k$ of Eq. (3-27) would be:

$$w_1=-3, \quad w_2=-1, \quad w_3=+3, \quad w_4=+1$$
Example 3-4

Note:
The bit rate:
\[ R = n / T_0 = \ell / T_s = 1 \text{ kbit/s} \]

The baud rate:
\[ D = N / T_0 = 1 / T_s = 0.5 \text{ kbaud} \]

The bit rate and the baud rate are related by:
\[ R = \ell D \]
L=4 multilevel signal

- The null bandwidth of the rectangular-pulse multilevel waveform, fig. 3-14a:
  \[ B = \frac{1}{T_s} = D = 500\text{Hz} \]

- The absolute bandwidth of the \( \sin(x)/x \)-pulse multilevel waveform, fig. 3-14b:
  \[ B = \frac{N}{2T_0} = \frac{D}{2} = 250\text{Hz} \]

- In general, an \( L \)-level multilevel signal would have \( 1/\ell \) the bandwidth of the corresponding binary signal, where \( \ell = \log_2(L) \).
3.5 Line codes and spectra
Why line codes must be used?

Properties of a line codes

- Self-synchronization
- A spectrum that is suitable for the channel
- Low probability of bit error
- Transmission bandwidth should be as small as possible
- Error detection capability
- ......
**classification of the Line codes**

- *Binary line coding*

  - **Two major categories:**
    - Return-to-zero (RZ)
    - Nonreturn-to-zero (NRZ)
the line code may be further classified according to the rule used to assign voltage levels to represent the binary data.

- Unipolar signaling

- Polar signaling

- Bipolar (pseudoternary) signaling

- Manchester signaling
Power spectra for binary line codes

- The PSD can be evaluated by
  - Deterministic technique
  - Stochastic technique
- Deterministic technique

For a particular data sequence:

\[
p_w(f) = \lim_{T \to \infty} \left( \frac{|W_T(f)|^2}{T} \right)
\]

For a periodic waveform:

\[
p(f) = \sum_{n=-\infty}^{n=\infty} |c_n|^2 \delta(f - nf_0)
\]

- The stochastic approach will be used to obtain the PSD of the line codes, because the line codes have a random data sequence (instead of a particular data sequence)
For a digital signal (or line code):

\[ s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s) \]

General expression for the PSD of a digital signal

\[ p_s(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k)e^{j2\pi kfT_s} \]

Where \( F(f) \leftrightarrow f(t) \), \( R(k) \) is the autocorrelation of the data:

\[ R(k) = \sum_{i=1}^{l} (a_n a_{n+k})_i P_i \]
**Power spectra for binary line codes**

- **Unipolar NRZ**

\[
P_{\text{unipolar NRZ}}(f) = \frac{A^2 T_b}{4} \left( \frac{\sin \pi f T_b}{\pi f T_b} \right)^2 \left[ 1 + \frac{1}{T_b} \delta(f) \right]
\]

**Advantages:** Easy to generate, only require one power supply.

**Disadvantages:** Waste of power, dc coupled circuits are needed
Advantages: BER is superior to that of other signaling

Disadvantages: Having a large PSD near dc; positive and negative power supply are required.
Power spectra for binary line codes

- **unipolar RZ**

\[
P_{\text{unipolar RZ}}(f) = \frac{A^2 T_b}{16} \left( \frac{\sin(\pi f T / 2)}{\pi f T / 2} \right)^2 \left[ 1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_b}) \right]
\]

- **Advantages:**
  - There is a discrete pulse at \( f = R \), it can be used for recovery of the clock signal.

- **Disadvantages:**
  - The first null bandwidth is twice that for NRZ signal
  - Requires 3 dB more power than polar signal for the same \( P_e \)
  - The spectrum is not negligible for frequency near dc
**Power spectra for binary line codes**

- **bipolar RZ**

\[
P_{\text{bipolar RZ}}(f) = \frac{A^2 T_b}{8} \left( \frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2} \right)^2 (1 - \cos(2\pi f T_b))
\]

\[
P_{\text{bipolar RZ}}(f) = \frac{A^2 T_b}{4} \left( \frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2} \right)^2 \sin^2(\pi f T_b)
\]
bipolar RZ

**Advantages:**
- Has a spectral null at dc, so ac coupled circuit may be used.
- The clock signal can be easily be extracted for the bipolar waveform.
- Has single-error detection capabilities.

**Disadvantages:**
- The receiver has to distinguish between three levels (+A, -A, and 0).
- Requires approximately 3 dB more power than polar signal for the same $P_e$. It has $3/2$ the error of unipolar signal.
Power spectra for binary line codes

Manchester NRZ

\[ p_{\text{Manchester NRZ}}(f) = A^2 T_b \left( \frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2} \right)^2 \sin^2(\pi f T_b / 2) \]

The PSD of line code (positive part of frequency)
Manchester NRZ

**Advantages:**
- Has a zero dc level
- A string of zeros will not cause a loss of the clocking signal

**Disadvantages:**
- The null bandwidth is twice that of the bipolar bandwidth
The spectrum is a function of the bit pattern (via the bit autocorrelation) as well as the pulse shape.

The general result for the PSD, Eq. (3-36), is valid for multilevel as well as binary signaling.
3.5.3 Differential coding
3.5.3 Differential coding

- **Goal:** to settle the question that the waveform is inverted.
- **Method:** differential coding is employed.

Encoded differential data are generated by

\[ e_n = d_n \oplus e_{n-1} \]

The received encoded data are decoded by

\[ \tilde{d}_n = \tilde{e}_n \oplus \tilde{e}_{n-1} \]
3.5.3 Differential coding

Encoded sequence
\[ e_n = d_n \oplus e_{n-1} \]

Decoded sequence
\[ d_n = e_n \oplus e_{n-1} \]

### Table 3-4: Example of Differential Coding

<table>
<thead>
<tr>
<th>Encoding</th>
<th>Decoding (with correct channel polarity)</th>
<th>Decoding (with inverted channel polarity)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input sequence</strong></td>
<td>(d_n)</td>
<td>(\tilde{e}_n) (correct polarity)</td>
</tr>
<tr>
<td><strong>Encoded sequence</strong></td>
<td>(e_n)</td>
<td>(1)</td>
</tr>
<tr>
<td><strong>Reference digit</strong></td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>Received sequence</strong></td>
<td>(\tilde{d}_n)</td>
<td>1</td>
</tr>
<tr>
<td><strong>Decoded sequence</strong></td>
<td>(\tilde{e}_n) (correct polarity)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Decoded sequence</strong></td>
<td>(\tilde{d}_n) (inverted polarity)</td>
<td>1</td>
</tr>
</tbody>
</table>
3.5.4 Eye patterns

The eye pattern provides the following information:

- The timing error allowed on the sampler at the receiver
- The sensitivity to timing error
- The noise margin
3.5.5 Regenerative repeaters

- In long-distance digital communication systems, repeaters are necessary to **amplify and “clean up”** the signal periodically.
- In the analog system, **linear amplifiers** with appropriate filters can be used.
- In the digital system, **Regenerative repeater** can be used.
3.5.6 Bit synchronization

- Digital communication usually need at least three types of synchronization signals:
  - **Bit sync**, to distinguish one bit interval from another
  - **Frame sync**, to distinguish groups of data
  - **Carrier sync**. For bandpass signaling with coherent detection

- Two kinds of ways to obtain synchronization signals:
  - directly from the corrupted signal
  - from a separate channel that is used only to transmit the sync. information
3.5.6 Bit synchronization

- The complexity of the bit synchronizer circuit depends on the sync properties of the line code.

- **Example:**
  - The methods to obtain the bit sync clock signal for a *unipolar RZ* code:
    - To use narrowband bandpass filter
    - To use the phase-locked loop (PLL)

- **Example:**
  - the bit synchronizer for a *polar NRZ* line code.
3.5.6 Bit synchronization

(c) Unipolar RZ

(b) Polar NRZ
3.5.6 Bit synchronization

- **Note:**
  - Unipolar, polar, and bipolar bit synchronizers will work only when there are a sufficient number of alternating 1’s and 0’s in the data.

- Long strings of all 1’s or 0’s must be prevented.
  - To use bit interleaving;
  - To use a completely different type of line code that does not require alternating data for bit synchronization.
Multilevel signaling provides reduced bandwidth compared with binary signaling.

A binary signaling can be converted to a multilevel polar NRZ signal:

\[
\ell \text{ bit D/A transfer:}
\]

Single polar NRZ
Binary input
\(w_1(t)\)
R bit/s

\[\ell \text{ bit D/A transfer} \]

Multilevel polar NRZ
L-level waveform output
\(w_2(t)\)
\(D \text{ symbol/s}=R/\ell\),
R bit/s
### Example: Three-bit DAC code

\[ \ell = 3, \quad L = 2^\ell = 8 \]

<table>
<thead>
<tr>
<th>Digital Code</th>
<th>Output</th>
<th>((a_n)_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>+7</td>
<td></td>
</tr>
<tr>
<td>001</td>
<td>+5</td>
<td></td>
</tr>
<tr>
<td>010</td>
<td>+3</td>
<td></td>
</tr>
<tr>
<td>011</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>101</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>111</td>
<td>-7</td>
<td></td>
</tr>
</tbody>
</table>
The baud rate

\[ D = \frac{1}{T_s} \]

\[ = \frac{1}{(3T_b)} \]

\[ = \frac{R}{3} \]

In general

\[ D = \frac{R}{\ell} \]

Bandwidth

\[ B \geq \frac{D}{2} \]
General expression for the PSD of a digital signal

\[ p_s(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k)e^{j2\pi kfT_s} \]

Where \( F(f) \leftrightarrow f(t) \), \( R(k) \) is the autocorrelation of the data:

\[ R(k) = \sum_{i=1}^{I} (a_n a_{n+k})_i P_i \]

Note:

The spectrum of the digital signal depend on two things:

1. The pulse shape used
2. Statistical properties of the data
For the where case $\ell = 3$, $T_s = 3T_b$, the rectangular pulse shape of width $3T_b$:

$$p_{w2}(f) = 63T_b \left( \frac{\sin 3\pi f T_b}{3\pi f T_b} \right)^2$$

the first null bandwidth for this multilevel polar NRZ signal is:

$$B_{null} = \frac{1}{3T_b} = \frac{R}{3}$$

----- one-third the bandwidth of the input binary signal
In general, for the case of $L = 2^\ell$ levels, the PSD of a multilevel polar NRZ signal with rectangular pulse shape:

$$P_{w2}(f) = K \left( \frac{\sin \ell \pi f T_b}{\ell \pi f T_b} \right)^2$$

The null bandwidth is

$$B_{null} = \frac{R}{\ell}$$
**Definition**: The spectral efficiency of a digital signal is given by the number of bits per second of data that can be supported by each hertz of bandwidth. That is, the spectral efficiency \( \eta \) is:

\[
\eta = \frac{R}{B}
\]

Where:  
- \( R \) is the data rate  
- \( B \) is the bandwidth
The spectral efficiency for multilevel polar NRZ signal

\[ \eta = \frac{R}{B} = \frac{R}{R/\ell} = \ell \quad (\text{bit} / \text{s}) / \text{Hz} \]

Where \( \ell \) is the number of bits used in the DAC.

**Note:**

\( \ell \) Cannot be increased without limit to an infinite efficiency, because it is limited by the signal-to-noise ratio.
The maximum possible spectral efficiency is limited by the channel noise if the error is to be small.

The maximum spectral efficiency $\eta_{\text{max}}$ is:

$$\eta = \frac{C}{B} = \log_2 \left( 1 + \frac{S}{N} \right)$$

To approach this maximum spectral efficiency, practical systems usually incorporate error correction coding and multilevel signaling.
## Spectral efficiency of Line codes

<table>
<thead>
<tr>
<th>Code type</th>
<th>First zero-point Bandwidth (Hz)</th>
<th>Frequency efficiency $\eta = \frac{R}{B}$ [(b/s)/Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unipolar NRZ</td>
<td>$R$</td>
<td>1</td>
</tr>
<tr>
<td>polarity NRZ</td>
<td>$R$</td>
<td>1</td>
</tr>
<tr>
<td>Unipolar RZ</td>
<td>$2R$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Bipolar RZ</td>
<td>$R$</td>
<td>1</td>
</tr>
<tr>
<td>Manchester NRZ</td>
<td>$2R$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Multilevel polarity NRZ</td>
<td>$R/\ell$</td>
<td>$\ell$</td>
</tr>
</tbody>
</table>