3.6 Intersymbol interference
3.6 Intersymbol interference

what is intersymbol interference and what cause ISI

1. The absolute bandwidth of rectangular multilevel pulses is infinite. The channels bandwidth is limited.

2. The channels property is not plat. Pulses are filtered improperly as they through channel, they will spread in time.
Intersymbol interference

Input waveform $w_{in}(t)$

Sample points (transmitter clock)

Individual impulse response

Sample point (receiver clock)

Received waveform $w_{out}(t)$ (pulse response sum)

Fig 3-23
How can we restrict the bandwidth and still not introduce ISI?

Nyquist discovered three different methods for pulse shaping that could be used to eliminate ISI.
The digital signaling system:

\[ w_{in}(t) \xrightarrow{\text{send filter}} h_{T}(t), H_{T}(f) \xrightarrow{\text{channel(filter)}} h_{C}(t), H_{C}(f) \xrightarrow{\text{receiver filter}} h_{R}(t), H_{R}(f) \rightarrow w_{out}(t) \]

recovered rounded pulse
(to sample and decode circuit)

The equivalent impulse response is

\[ h_{e}(t) = h(t) \ast h_{T}(t) \ast h_{C}(t) \ast h_{R}(t) \]

The equivalent system transfer function:

\[ H_{e}(f) = H(f) \ast H_{T}(f) \ast H_{C}(f) \ast H_{R}(f) \]
If the equivalent system impulse response satisfies the condition

\[ h_e(kT_s + \tau) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases} \]

It will eliminating ISI

Where:  
- C is a nonzero constant,
- K is an integer,
- \( T_s \) is the symbol (sample) clocking period,
- \( \tau \) is the offset in the receiver sampling clock times, compared with the clock times of the input symbols.
If we choose a \((\sin x)/x\) function for \(h_e(t)\), the impulse response satisfies Nyquist’s first criterion for zero ISI.

\[
h_e(t) = \frac{\sin \pi f_s t}{\pi f_s t}
\]

where \(f_s = 1 / T_s\)
Nyquist’s First Method (zero ISI)

- If the transmit and receive filters are designed so that the overall transfer function is

\[ H_e(f) = \frac{1}{f_s} \prod \left( \frac{f}{f_s} \right) \]

There will be no ISI, furthermore, the absolute bandwidth of this transfer function is \( B = f_s / 2 \)

**Difficulties:**

- \( H_e(f) \) is difficult to approximate because of the steep skirts in the filter transfer function
- The synchronization of the clock in the decoding sampling circuit has to be almost perfect.
Because of these difficulties, we are forced to consider other pulse shapes.

The idea is to find pulse shapes that go through zero at adjacent sampling points, and yet have an envelope that decays much faster than $1/x$, so that clock jitter in the sampling times does not cause appreciable ISI.

Solution: Raised cosine-rolloff Nyquist filter.
The raised cosine-rolloff Nyquist filter has the transfer function

$$H_e(f) = \begin{cases} 
1 &, \quad |f| < f_1 \\
\frac{1}{2} \left[ 1 + \cos \left( \frac{\pi}{2f_\Delta} \left( |f| - f_1 \right) \right) \right] &, \quad f_1 < |f| < B \\
0 &, \quad |f| > B 
\end{cases}$$

Where $B$ is the absolute bandwidth

$$f_\Delta = B - f_0, \quad f_1 = f_0 - f_\Delta,$$

$f_0$ is 6dB bandwidth of the filter.

The rolloff factor is defined:

$$r = \frac{f_\Delta}{f_0}.$$
The corresponding **impulse response** is:

\[
h_e(t) = F^{-1}[H_e(f)] = 2f_0 \left( \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \right) \left[ \frac{\cos(2\pi f_\Delta t)}{1 - (4f_\Delta t)^2} \right]
\]

**Frequency and time response for different rolloff factor**

As the absolute bandwidth is increased (r=0.5 or 1):

1. The filtering requirements are relaxed.
2. The clock timing requirements are also relaxed.
The 6-dB bandwidth of the raised cosine-rolloff filter $f_0$ is designed to be half the symbol (baud) rate.

The baud rate that the raised cosine-rolloff system can support without ISI

\[
T_s = \frac{1}{2f_0} \quad D = \frac{1}{T_s} = 2f_0 \quad f_0 = \frac{D}{2}
\]

\[
r = f_\Delta / f_0 \quad f_0 = \frac{B}{1+r} \quad D = \frac{2B}{1+r}
\]

$r_\Delta = B - f_0$
Example 3-1

Assume that a binary digital signal, with Polar NRZ signaling, is pass through a communication system with a raised cosine-rolloff filtering characteristic.

Let the rolloff factor be 0.25. the bit rate of the digital signal is 64 kbit/s.

Determine the absolute bandwidth of the filtered digital signal.
The raised cosine-rolloff filter is only one of a more general class of filters that satisfy Nyquist’s first criterion.

The general class of filters that satisfy Nyquist’s first criterion---- Nyquist filter.
A filter is said to be a Nyquist filter if the effective transfer function is

\[ H_e(f) = \begin{cases} \prod \left( \frac{f}{2f_0} \right) + Y(f), & |f| < 2f_0 \\ 0, & f \text{ elsewhere} \end{cases} \]

where \( Y(f) \) is a real function that is even symmetric about \( f=0 \)

\[ Y(-f) = Y(f), \quad |f| < 2f_0 \]

And \( Y(f) \) is odd symmetric about \( f=f_0 \)

\[ Y(-f+f_0) = -Y(f+f_0) \quad , \quad |f| < f_0 \]

Then there will be no ISI at the system output if the symbol rate is

\[ D = f_s = 2f_0 \]
Nnyquist filter

\[ H_e(f) = \begin{cases} 
\prod \left( \frac{f}{2f_0} \right) + Y(f), & |f| < 2f_0 \\
0, & f \text{ elsewhere}
\end{cases} \]
Nyquist second and third methods for control of ISI

- **Nyquist’s second method (ISI control)**
  - Allows some ISI to be introduced in a controlled way, so that it can be canceled out the receiver and the data can be recovered without error if no noise is present.

- **Nyquist’s third method (ISI control)**
  - The effect of ISI is eliminated by choosing $h_e(t)$: the area under $h_e(t)$ pulse within the desired symbol interval, $T_s$, is not zero, but the areas under $h_e(t)$ in adjacent symbol intervals are zero.
3.7 Differential pulse code modulation
Differential pulse code modulation

The reason of we use DPCM

- There is a lot of redundancy in the signal samples.
- The bandwidth and the dynamic range of a PCM system are wasted

Solution

To transmit the difference in adjacent sample values. That is, to use Differential pulse code modulation (DPCM)

Method

To use prediction filter

\[ x_n = \sum_{i=1}^{k} a_i x_{n-i} \]
The prediction filter may be realized by using a tapped delay line to form a transversal filter.

The output samples are

\[ z(nT_s) = \sum_{l=1}^{K} a_l y(nT_s - lT_s) \]

In simplified notation:

\[ Z_n = \sum_{l=1}^{K} a_l y_{n-l} \]
Differential pulse code modulation

The first configuration

- using prediction from samples of input signal
Differential pulse code modulation

The second configuration

- using prediction from quantized differential signal
DPCM, like PCM, follows the 6-dB rule

\[
\left( \frac{S}{n} \right)_{dB} = 6.02n + \alpha
\]

Unlike companded PCM, the \( \alpha \) for DPCM varies over a wide range, depending on the Properties of the input analog signal. For DPCM speech: \(-3 < \alpha < 15\)

The DPCM performance may be compared with that for PCM

For the same SNR, DPCM could require 3 or 4 fewer bits per sample than companded PCM.
A 32-Kbits/s DPCM CCITT standard:
- To use 4-bit quantization at an 8-Ksample/s rate for encoding 3.2-KHz bandwidth VF signals.

A 64-Kbits/s DPCM CCITT standard:
- To use 4-bit quantization and 16-Ksample/s for encoding audio signals that have a 7-KHz bandwidth.
3.8 Delta modulation
**Delta modulation**

- **DM—Delta Modulation. It is a special case of DPCM.**

**Characteristics:**

- There are only two quantizing levels
- Only one bit is transmitted per sample.
- ……
DM system waveforms

(a) Analog Input and Accumulator Output Waveforms

(b) Delta Modulation Waveform
Granular noise & slope overload noise

Slope overload noise

\[ \delta \]

Granular noise

Slope overload noise will decrease as \( \delta \) increase.

Granular noise will decrease as \( \delta \) decrease.
Granular noise & slope overload noise
Example 3-5: Design of a DM system.

**Problem:** find the step size $\delta$ required to prevent slope overload noise for the case when the input signal is a sine wave.

$$w(t) = A \sin \omega_a t$$
the granular noise power in the analog output signal band:

\[ N = \langle n^2 \rangle = \int_{-B}^{B} P_n(f)df = \frac{\delta^2 B}{3 f_s} \]

From eq.(3-84), with equality:

\[ N = \frac{4 \pi^2 A^2 f_a^2 B}{3 f_s^2} \]

The signal power is (for a sine-wave test signal)

\[ S = \langle w^2(t) \rangle = \frac{A^2}{2\pi} \]
The resulting average signal-to-quantizing noise ratio:

\[
\left( \frac{S}{N} \right)_{\text{out}} = \frac{3}{8\pi^2} \frac{f_s^3}{f_a^2 B}
\]

- \( f_s \) --- the DM sampling frequency
- \( f_a \) --- the frequency of the sinusoidal input
- \( B \) --- the bandwidth of the receiving system

Attention: This Eq. is valid only for sinusoidal-type signal
Adaptive Delta modulation ADM:
the step size vary as a function of time as the input waveform changes.

When signal ↑ → δ ↑

When signal ↓ → δ ↓
Method 1

- The step size may be adapted by examining the DM pulses at the transmitter output.

- When the DM pulses consist of a string of pulses with the same polarity, the step size is increased until the DM pulses begin to alternate in polarity, then the step size is decreased, and so on.
**step-size Algorithm:**

<table>
<thead>
<tr>
<th>Data Sequence</th>
<th>Number of Successive Binary 1’s or 0’s</th>
<th>Step-size Algorithm $f(d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>x x 0 1</td>
<td>1</td>
<td>$\delta$</td>
</tr>
<tr>
<td>x 0 1 1</td>
<td>2</td>
<td>$\delta$</td>
</tr>
<tr>
<td>0 1 1 1</td>
<td>3</td>
<td>$2 \delta$</td>
</tr>
<tr>
<td>1 1 1 1 1</td>
<td>4</td>
<td>$4 \delta$</td>
</tr>
</tbody>
</table>

X: don’t care
continuously variable slope delta modulation (CVSD)

Method 2

- CVSD is another variation of ADM

- An integrator (instead of accumulator) is used, so that \( z(t) \) is made continuously variable

Product

- The Motorola MC34115
- The Motorola MC3418
Question

Which is better, PCM or DM?

The answer depends on the criterion used for comparison and the type of message.

- To have a relatively simple, low-cost system, DM may be the best
- To have a high output SNR, PCM probably the best
- To interface existing equipment, compatibility, PCM has the advantage.
3.9 **Time-Division Multiplexing (TDM)**
Time-Division Multiplexing (TDM)

Why we must use TDM?

- **Aims:** to make use of the channel bandwidth to achieve high spectral efficiency

What is the TDM?

- TDM (Time-division multiplexing) is the time interleaving of samples from several sources so that the information from these sources can be transmitted serially over a single communication channel.
Time-Division Multiplexing (TDM)

three analog sources are multiplexed over a PCM system.
The pulse width of the TDM PAM:

$$\frac{T_s}{3} = \frac{1}{3f_s}$$

The pulse width of the TDM PCM:

$$\frac{T_s}{3n} = \frac{1}{3nf_s}$$
Aims of the frame sync. : To make the received multiplexed data can be sorted and directed to the appropriate output channel at the TDM receiver.

The frame sync. Signal can be provided to the receiver demultiplexer by:

- Sending a frame sync signal over a separate channel
- Deriving the frame sync from the TDM signal itself
Frame synchronization word:
A segmented bits data stream which obeys some rules. Usually, it should be unique in the data stream, or at least, the appear probability is very small.

\[
\begin{array}{cccccccc}
\cdots & s_1 & s_2 & \cdots & s_k & Ch. 1 & Ch. 2 & \cdots & Ch. N & s_1 & s_2 & \cdots & s_k & \cdots
\end{array}
\]
Example 3.6

- Design a time-division multiplexer that will accommodate 11 sources, assume that the sources have the following specifications:
  
  - Source 1. analog, 2-kHz bandwidth
  - Source 2. analog, 4-kHz bandwidth
  - Source 3. analog, 2-kHz bandwidth
  - Sources 4-11. digital, synchronous at 7200 bits/s.
Example

Time-Division Multiplexing (TDM)

From source 1
2 kHz, analog
TDM PAM signal
16 ksamples/sec
4 bit A/D
64 kb/sec

From source 2
4 kHz, analog
$f_1 = 4$ kHz

From source 3
2 kHz, analog

From source 4
7.2 kb/sec, digital
Pulse stuffing
8 kb/sec

From source 5
7.2 kb/sec, digital
Pulse stuffing
$f_2 = 8$ kHz

From source 11
7.2 kb/sec, digital
Pulse stuffing
8 kb/sec

TDM PCM Output signal
128 kb/sec
The preceding example illustrates the main advantage of TDM:

It can easily accommodate both analog and digital sources.

Unfortunately, when analog signals are converted to digital signals without redundancy reduction, they consume a great deal of digital system capacity.
TDM hierarchy

Two categories:

- **TDM used in digital computer system**
  
The output rate has been standardized to 1.2, 2.4, 3.6, 4.8, 7.2, 9.6, 14.4, 19.2, 28.8 kb/s. and to 10 and 100 to 1000Mb/s, 10Gb/s.

- **TDM used by common carrier**
  
  - North American digital TDM hierarchy
  - Europe digital TDM hierarchy (CCITT TDM)
North American digital TDM hierarchy: (T1 TDM system)
24-VF analog telephone signals are converted to a DS-1 (1.544 Mbit/s) data stream.

The sampling rate used on each of the 24-VF analog signals is 8 kHz.

Each analog sample is encoded into an 8-bit PCM word.

There are $8 \times 24 = 192$ bits of data, plus one bit is added for frame synchronization, yielding a total of 193 bits per frame.
Europe digital TDM hierarchy:

(CCITT TDM standard)