Chapter 4
Bandpass signaling principles and circuits
In this section, we will answer the questions:

- What is a general representation for bandpass digital and analog signals?
- How do we represent a modulated signal?
- How do we represent bandpass noise?
4.1 Complex envelope representation
4.1 Complex envelope representation

Modulation

- The process of imparting the source information onto a bandpass signal with a carrier frequency $f_c$ by the introduction of amplitude or phase perturbations or both.

The modulation may be visualized as a mapping operation that maps the source information onto the bandpass signal.
4.1 Complex envelope representation

The modulated signals representation

\[ v(t) = R(t) \cos(\omega_c t + \theta(t)) \]

\[ v(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t \]

\[ x(t) = R(t) \cos \theta(t) \]

\[ y(t) = R(t) \sin \theta(t) \]

\[ v(t) = \text{Re}\left\{g(t)e^{j\omega_c t}\right\} \]

\[ g(t) = R(t)e^{j\theta(t)} = R(t)\left[\cos \theta(t) + j \sin \theta(t)\right] = x(t) + jy(t) \]
4.1 Complex envelope representation

**Theorem**

Any physical bandpass waveform can be represented by

\[ v(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \]

Two other equivalent representations:

\[ v(t) = R(t) \cos(\omega_c t + \theta(t)) \]

\[ v(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t \]
4.2 Representation of modulated signals
The modulated signal is represented by

\[ s(t) = \text{Re}\left\{ g(t)e^{j\omega_c t} \right\} \]

where the complex envelope \( g(t) \) is a function of the modulating signal \( m(t) \):

\[ g(t) = g[m(t)] \]

\( g[.\text{]} \) performs a mapping operation on \( m(t) \)
Properties of complex envelope $g(t)$:

$$g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}$$

$$x(t) = \text{Re}\{g(t)\} = R(t)\cos\theta(t)$$

$$y(t) = \text{Im}\{g(t)\} = R(t)\sin\theta(t)$$

$$R(t) = |g(t)| = \sqrt{x^2(t) + y^2(t)}$$

$$\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$
4.3 Spectrum of bandpass signals
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A bandpass waveform \( v(t) \) can be represented as:

\[
v(t) = \text{Re}\left\{ g(t) e^{j\omega t} \right\}
\]

Then, if \( v(t) \leftrightarrow V(f) \) and \( g(t) \leftrightarrow G(f) \), the spectrum of the bandpass waveform is:

\[
V(f) = \frac{1}{2} \left[ G(f - f_c) + G^*(-f - f_c) \right]
\]

The PSD of the waveform is:

\[
p_v(f) = \frac{1}{4} \left[ p_g(f - f_c) + p_g(-f - f_c) \right]
\]
4.3 Spectrum of bandpass signals

The PSD of the waveform is

\[ p_v(f) = \frac{1}{4} \left[ p_g(f - f_c) + p_g(-f - f_c) \right] \]

The total average normalized power of a bandpass waveform \(v(t)\) is

\[
P_v = \langle v^2(t) \rangle \\
= \int_{-\infty}^{+\infty} p_v(f) df \\
= R_v(0) \\
= \frac{1}{2} \langle |g(t)|^2 \rangle
\]
The peak envelope power (PEP) is the average power that would be obtained if $|g(t)|$ were to be held constant at its peak value.

The normalized PEP is given by

$$P_{PEP} = \frac{1}{2} \left[ \max |g(t)| \right]^2$$
**Example**

Evaluate the magnitude spectrum for an amplitude-modulated (AM) signal.
4.5 Bandpass filtering and linear distortion
Bandpass filtering and linear distortion

Equivalent Low-pass filter

\[ v_1(t) = \text{Re} \left[ g_1(t)e^{j\omega_c t} \right] \]

\[ h_1(t) = \text{Re} \left[ k_1(t)e^{j\omega_c t} \right] \]

\[ H(f) = \frac{1}{2} K (f - f_c) + \frac{1}{2} K^* (-f - f_c) \]

(a) Bandpass Filter

(b) Typical Bandpass Filter Frequency Response
**Theorem**

- The complex envelopes for the input, output, and impulse response of a bandpass filter are related by

\[
\frac{1}{2} g_2(t) = \frac{1}{2} g_1(t) * \frac{1}{2} k(t)
\]

where \(g_1(t)\) is the complex envelope of the input and \(k(t)\) is the complex envelope of the impulse response. It also follows that

\[
\frac{1}{2} G_2(f) = \frac{1}{2} G_1(f) \cdot \frac{1}{2} K(f)
\]
Equivalent Low-pass filter

(c) Equivalent (Complex Impulse Response) Low-pass Filter

(d) Typical Equivalent Low-pass Filter Frequency Response
To have no distortion at the output of a linear time-invariant system, two requirements must be satisfied:

- **The amplitude response is flat.** That is,
  \[ |H(f)| = A \quad \text{A: constant} \]

- **The phase response is a linear function of frequency.** That is,
  \[ \theta(f) = \angle H(f) = -2\pi f T_d \]
For distortionless transmission of bandpass signals, the channel transfer function \( H(f) = |H(f)|e^{j\theta(f)} \) needs to satisfy the following requirements:

- The amplitude response is constant.
  \[
  |H(f)| = A \tag{4-27a}
  \]

- The derivative of the phase response is a constant.
  \[
  -\frac{1}{2\pi} \frac{d\theta(f)}{df} = T_g \tag{4-27b}
  \]
Linear distortion

Note:
The Eqs. (4-27a) and (4-27b) are only **sufficient requirements** for distortionless transmission of bandpass signals.
The output bandpass signal can be described by

\[ v_2(t) = Ax(t - T_g) \cos[\omega_c (T - T_d)] - Ay(t - T_g) \sin[\omega_c (t - T_d)] \]

The bandpass filter delays the input complex envelope (i.e., the input information) by \( T_g \), whereas the carrier is delayed by \( T_d \).

**Note:** \( T_g \) will differ from \( T_d \), unless \( \theta_0 \) happens to be zero.
The general requirements for distortionless transmission of either baseband or bandpass signals are given by Eqs. (2-150a) and (2-150b).

However, for the bandpass case, Eq. (2-150b) is overly restrictive and may be replaced by Eq. (4-27b).

For distortionless bandpass transmission, it is only necessary to have a transfer function with a constant amplitude and a constant phase derivative over the bandwidth of the signal.
4.6 Bandpass sampling theorem
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Bandpass sampling Theorem

- If a (real) bandpass waveform has a nonzero spectrum only over the frequency interval \( f_1 < |f| < f_2 \), where the transmission bandwidth \( B_T \) is taken to be the absolute bandwidth \( B_T = f_2 - f_1 \), then the waveform may be reproduced form sample values if the sampling rate is

\[
f_s \geq 2B_T
\]
4.6 Bandpass sampling theorem

- **Proof.**
- the quadrature bandpass representation is
  \[ v(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t \]
- sampling theorem

\[ v(t) = \sum_{n=-\infty}^{\infty} \left[ x\left(\frac{n}{f_b}\right) \cos \omega_c t - y\left(\frac{n}{f_b}\right) \sin \omega_c t \right] \frac{\sin\{\pi f_b (t-(n/f_b))\}}{\pi f_b (t-(n/f_b))} \]

- For the general case, where the \( x (n/ f_b) \) and \( y (n/ f_b) \) samples are independent, **two real samples are obtained for each value of \( n \)**, so that the overall sampling rate for \( v(t) \) is \( f_s = 2 f_b \geq 2 B_T \).
Example SA 4.5

The signal $s(t)$ is to be sampled by using any one of three methods, for each of three sampling methods, determine the minimum sampling frequency required.

![Diagram of three methods for sampling bandpass signals.](image)

*Figure 4-32 Three methods for sampling bandpass signals.*
Bandpass dimensionality theorem

Assume that a bandpass waveform has a nonzero spectrum only over the frequency interval $f_1 < |f| < f_2$, where the transmission bandwidth $B_T$ is taken to be the absolute bandwidth given by $B_T = f_2 - f_1$ and $B_T << f_1$, the waveform may be completely specified over a $T_0$-second interval by

$$N = 2 B_T T_0$$

independent pieces of information. $N$ is said to be the number of dimensions required to specify the waveform.
4.7 Received signal plus noise
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The signal out of the transmitter is

\[ s(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \]

The received signal plus noise is

\[ r(t) = s(t) * h(t) + n(t) \]

If the channel is distortionless, \( H(f) = Ae^{j(2\pi T_g + \theta_0)} = (Ae^{j\theta_0})e^{-j2\pi T_g} \)

the signal plus noise at the receiver input is

\[ r(t) = \text{Re}\{Ag(t - Tg)e^{j(\omega_c t + \theta(f))}\} + n(t) \]
If the receiver circuits are designed to make errors due to the channel group delay \((T_g)\) and \(\theta(f)\) effects negligible, we can consider the signal plus noise at the receiver input to be

\[
r(t) = \text{Re}\left\{g(t)e^{j\omega_ct}\right\} + n(t)
\]

where the effects of channel filtering, if any, are included by some modification of the complex envelope \(g(t)\).